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Is There A Lunar Influence on Search and Rescue Incidents?

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We examine an 11 year window of search and rescue (SAR) incidents within a large Canadian region to see if there are correlations with lunar phase and distance, to highlight a novel and rich dataset for lunacy effect research and to see whether SAR facilities need to adjust their ready-to-respond postures accordingly. Using celestial mechanics, each incident is assigned a distance and phase value, and we perform statistical analyses on these two measures using derived probability density functions. There is no correlation with any particular phase or distance. However, a subclass of incidents have a statistically significant correlation to a *range* of phases, although the increase may not be enough to influence ready-to-respond postures.

Key Words: Lunacy Effect, Search and Rescue, Celestial Mechanics, Kolmogorov-Smirnov Statistics, Chi-Square Statistic, Maritime, Interpolation Methods

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Introduction

It has been widely debated whether the moon has a cyclical influence on human behavior. Many of the arguments on both sides are quite anecdotal as clearly demonstrated by the sheer volume of unsubstantiated discussions that can be found if one types "lunar phase" with "bar fights", "mental health" or "emergency room" in any internet search engine. However, there are many formal and rigorous studies looking to correlate the effects of the lunar cycle on human behavior. Perhaps one of the earliest known such study was conducted by John Haslam in 1809, wherein he tracked patient behavior as a function of lunar phase for over two years at the Bethlem Hospital in London (Haslam, 1809; Howard, 1989). Haslam observed no evidence of "lunacy" in the patients although reported that there was a noticeable effect on the staff who "without waiting for any display of increased turbulence on the part of the patient, he had bound, chained, flogged, and deprived these miserable people of food, according as he discovered the moon's age by the almanack [sic]" (Haslam, 1809, p. 216).

Kelly, Rotton and Culver conducted a comprehensive series of meta-analyses from 1985 to 1986 (Kelly et al., 1985-1986; Rotton & Kelly, 1985a,b), where they re-visited thirty-seven independent studies and have shown that there exists no significant effect on phases of the moon on any of: homicide rates, traffic incidents, crisis calls to police or fire stations, domestic violence, suicide rates, major disasters, assassinations, kidnappings, violence in prisons, frequency of assaults, psychiatric admissions, agitated behavior by nursing home residents, aggression by hockey players, gunshot wounds, emergency room admissions and stabbings (Kelly et al., 1985-1986; Rotton & Kelly, 1985a,b).

These meta-analyses reveal a broad range of human-related phenomena which have been studied to find correlations with the lunar cycle. We sample a few here to exemplify this broad range. One of the more well-known posited positive-correlation studies examined by Kelly et al. (1985-1986) was that by A. Lieber & Sherin (1972) which proposed a correlation between lunar cycle and the incidence of homicides in two Florida counties: Dade and Cuyahoga. This study was included later in Lieber's 1978 book (A. L. Lieber, 1978). Unfortunately for the study and the book, the statistical methods within it were shown to be invalid with explicit examples by Abell (1979). Abell went further to invalidate the claims of the book and the earlier study on the basis that it was "very bad science" due to poor statistical methods, to lack of presenting sufficient data for scrutiny, to a lack of consistent treatment of data between the Dade County and Cuyahoga County homicide incidents, to biased comments by Lieber such as the one where he suggests "science of having a prejudice against a lunar effect" (Abell, 1979) and to self-contradicting assertions ("Lieber implies that the effect is obvious and that everyone is aware...but then says that it is a small effect that requires large samples to reveal" (Abell, 1979)). Similar shortcomings of Lieber and Sherin's study were discussed in detail by several other independent studies (Kelly et al., 1985-1986; Sanduleak, 1985; Berman, 2003; Iosif & Ballon, 2005).

In 2004, Román et al. found a correlation to lunar phase with patients admitted with gastrointestinal bleeding. Researchers concluded that their analysis suggests an increase in the number of admissions for gastrointestinal bleeding during the full moon (Román et al., 2004), particularly for men and for variceal haemorrhaging. However, although the authors establish a correlation they do caution that "the wide variation in the number of admissions throughout the lunar cycle...could limit analysis and interpretation of results" and that "further studies are needed to...ascertain whether the lunar effect is, in fact, a myth or a reality" (Román et al., 2004).

In 2008 Baxendale and Fisher conducted a study to correlate lunar phase with the frequency of epileptic seizures. Although a statistically significant correlation was initially found, it later disappeared when researchers controlled for local clarity of the night sky (Baxendale & Fisher, 2008) suggesting a correlation between epileptic seizures and night-time illumination rather than lunar phase.

Abell & Greenspan (1979b,a) looked at 12,000 live and dead births at UCLA hospital for 51 lunar months and found no correlation with lunar phase, despite the opinions of the nursing staff there. This study was undertaken when a study by Menaker and Menaker noted such a correlation concerning New York births (Menaker & Menaker, 1959). Although Abell and Greenspan's study did not refute the findings found by Menaker and Menaker, their results indicate that a correlation is certainly not universal, leaving speculations as to why there was a statistical significance in the New York study.

In 2000, Barr completed a study which correlated the effect of lunar phase on 100 mentally ill patients during a 30-month period, using analysis of variance on a set of meta-data related to indicators for quality of life. The study showed a correlation with a full moon only in cases of patients with schizophrenia. That is, he implies that it may be the sensitivity to insomnia associated with schizophrenia that is the cause of the aberrant behavior that would naturally peak around the full moon because of the additional light that the moon would provide, rather than the moon in general affecting human behavior. Barr, however, recommended that such studies ought to be replaced in the future with direct, face-to-face measures rather than proxy indicators (admission times, etc.) and that they should be conducted with those diagnosed with schizophrenia rather than those with mood disorders.

In 2013 Belleville et al. (2013) looked for seasonal and lunar cycles on anxiety and mood disorders in patients complaining of chest pains in two Montreal hospital emergency rooms over a three year span. Although they found seasonal correlations, they concluded that there were no correlations with the lunar cycle.

Clearly, this is an interesting topic which continues to persist. As Barr (2000) states the "search for evidence of a lunar effect continues and should not be considered an entirely theoretical endeavour with little practical application." To that end, we look at whether there is any lunar correlation with another set of human-related phenomena: search and rescue incidents. If there were a lunar influence on human behaviour which could lead to increased incident rates in emergency rooms, then one would expect to see a similar effect in search and rescue incidents. To our knowledge, there have been no studies to date which have formally looked at whether there is such a correlation. There is a definite practical reason for investigating this question. In Canada, the search and rescue facilities have ready-to-respond times which depend on the time of day, whether it is a weekend or weekday, as well as a seasonal variation. If there is any strong lunar correlation then these facilities should adjust their ready-to-respond times accordingly.

The Canadian Coast Guard's *System of Information for Search and Rescue* (SISAR) database contains information about all search and rescue (SAR) incidents which have occurred within Canada since 1994. Because this database contains the location and time of all incidents which generated a SAR request, it is relatively simple to determine the phase of the moon at the time of incident. This report endeavors to explore a substantial subset of this database to see if a correlation with lunar phase or distance can be found. Not only would this be an exercise of adding more information to the compendium of data supporting or refuting this topic, but if a correlation is found this longitudinal study would be instrumental within the SAR communities in adjusting response levels to include a lunar component. In the literature there is often the perception that the lunar effect on human behavior may have its roots in the tidal effect (e.g., A. L. Lieber (1978)). Although Abell (1979) provides sound physics-based arguments as to why tidal effects ought to be negligible over the physical scale of human dimensions, we intend to test that hypothesis with empirical data.

It is prudent to note a lunar correlation does not necessarily imply proof of a change in human behavior due to a lunar phase. For example, if there were a noted increase in maritime SAR incidents near a full moon, it might simply be due to the fact that more people are on the water during a full moon because of the extended visibility that the moon offers; more boating would typically yield more boating accidents. On the other hand, a lack of correlation *would* suggest the moon does not cause behavioral changes in people. The concept that it could do so and lead to increased disruptive behavior in mental health institutions, increased visits to emergency rooms and increased bar fights, but *not* to those who require SAR seems rather unlikely and skewed.

Finally, most of the studies looking into this phenomenon typically bin the data into days of the lunar phase. For instance, Abell & Greenspan (1979b) took any births occurring on the day of the full moon as "day 15" (the lunar phase has a synodic period of 29 to 30 days). In this study, we use an alternative approach and assign each SAR event with a specific fractional lunar phase value according to date and time, and then conduct the statistical analysis on these phase values. In order to do so, we make use of the lunar phase ephemeris available from the NASA website (NASA Lunar Phase Ephemeris, 2013). This comes with a warning of how to correctly interpolate between known phase values in the ephemeris due to celestial mechanics and we provide a fair amount of detail on correct interpolation methods.

This paper is organized as follows. In the next section the SISAR database is described and the portion used in this study is explained. Following that, the necessary celestial mechanics is discussed to identify the lunar phase at the time of the SAR incident. The section that follows analyzes the data once lunar phase is incorporated and the final section is reserved for conclusions and discussion.

SISAR Database

As stated in the Introduction, the SISAR database is a repository of all of the information collected on SAR incidents that have occurred in Canada since 1994. Each incident is associated with a date, time (specified in co-ordinated universal time, UTC) and location (specified by latitude and longitude), as well as SAR response data, including the name of the responding aircraft and its location, the time to respond, the action taken (e.g. search, hoisting, evacuation, recovery), the incident classification and the incident type. The incident classification (represented by the letters A, M, C, H, U; the meanings of which are given below) and type (represented by the numbers 1, 2, 3, 4) are defined in a Canadian Coast Guard Report (2001). The incident classification falls into five categories provided in Table 1, and the severity falls into one of four types shown in Table 2.

Canada is divided into three geographical areas for SAR and each area is monitored by a Joint Rescue Co-ordination Center (JRCC). Note that the JRCC is a co-ordination

Table 1: *SAR Classification*

Classification	Meaning
Aeronautical	Involving an aircraft.
Maritime	Involving a vessel or a person aboard a vessel, including the evacuation of a person from a vessel.
Civilian	Representing requests for assistance from some civilian authority, such as the police force; this class was eliminated in 1999 in favor of the more comprehensive humanitarian class.
Humanitarian	Representing requests that involve neither an aircraft nor a vessel.
Unknown	Representing incidents for which the source is never traced.

Table 2: *SAR Severity*

Severity	Meaning
In Distress	A vessel or a person is threatened by grave and imminent danger and requires immediate assistance (i.e. a life-threatening situation is judged to be present or close at hand at some point during the incident).
Potential Distress	The potential exists for a distress incident if timely action is not taken; i.e., immediate responses are required to stabilize a situation in order to prevent distress.
Non-Distress	No distress or perceived appreciable risk to life is apparent; this category includes general calls for assistance.
False alarms and hoaxes	Situations that cause the SAR system to react which proves to be unjustified or fabricated, such as a mistaken report of a flare.

office only; assets deployed to SAR incidents can come from several locations throughout Canada and not necessarily from the same location as the JRCC. The data for analysis in the current study represents those SAR incidents for the Canadian interior, monitored by JRCC Trenton, for the eleven year period of 1994 to 2004 (the validated data for this window of time was available to the authors). This geographical area, which includes most of Canada with the exclusion of the East and West Coast regions, is shown in Figure 1. This set was chosen for its geographical expanse. The number of SAR incidents within each SAR classification and severity level are shown in Table 3 for this area and for this 11 year window. Note that although we have separated "Civilian" from "Humanitarian" in this table, these classifications are combined in our analysis since the "Civilian" classification was merged with "Humanitarian" in 1999.

Methods

This study will be looking at the lunar phase at which the incident occurs, in part by comparing the incident's date and time to a lunar phase ephemeris (based on observational

Figure 1: *Geographical area from which the SISAR data were sampled.*

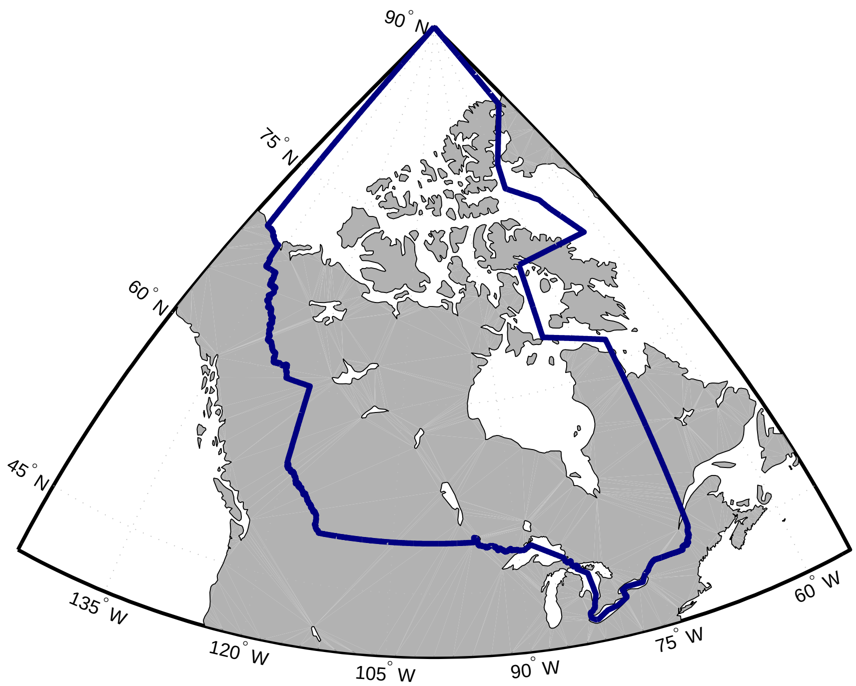


Table 3: *Distribution of All 16,315 SAR Incidents within the SISAR Data*

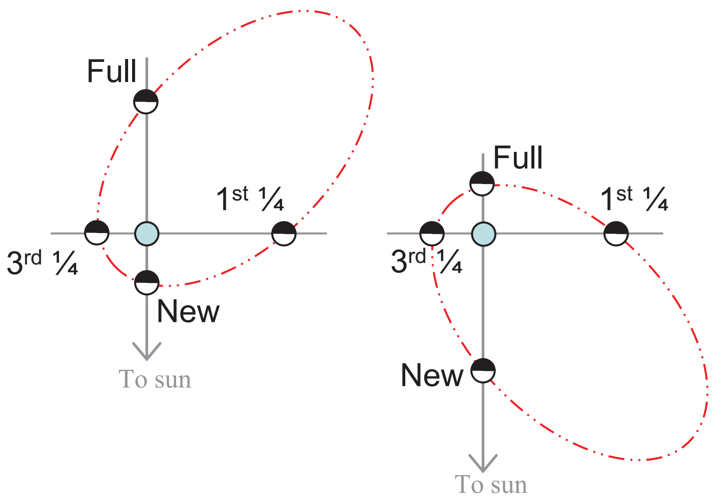
	In Distress	Potential Distress	Non-Distress	False Alarms and Hoaxes	
Aeronautical	351	119	93	1,488	*2,051*
Maritime	701	747	8,199	1,965	*11,612*
Civilian	71	34	152	98	*355*
Humanitarian	341	202	429	192	*1,164*
Unknown				1,133	*1,133*
	1,464	*1,220*	*8,873*	*4,877*	*16,315*

Grey italic numbers are the totals across the rows and columns.

data and not theoretical models) found on NASA's website (NASA Lunar Phase Ephemeris, 2013). Unfortunately, this ephemeris only provides times for four distinct phases of the moon (new, first quarter, full, third quarter) and so interpolation is required. For example, an incident may have occurred on 8 September 1994 at 14:30 universal time (UT) which is between a new moon (5 September 1994 at 18:33 UT) and the first quarter (12 September 1994 at 11:34 UT), and so the intermediate lunar phase needs to be obtained through interpolation. In the following derivation, the earth-moon system is considered, but the mechanics described herein applies to any object moving about a central body, the motion of which is due to gravity.

Interpolation must be carefully considered here, primarily since the moon's orbit is approximated by an ellipse and not a circle (albeit an ellipse with a small eccentricity, 0.054891; the minor axis differs from the major axis by about 0.15%), and thus linear interpolation of phase through time will yield spurious results. To demonstrate, consider Figure 2, in which the earth-moon system is shown at two different times in the year. For illustrative purposes, the elliptical orbit is exaggerated.

Figure 2: *Lunar Phase Overlay on the Lunar Elliptical Orbit (Exaggerated)*



Clearly, interpolating between, say, the new moon and the first quarter could yield different results depending on the ellipse's orientation and on the manner in which the interpolation is performed. A second ephemeris will thus be used which provides the lunar apogee (farthest) and perigee (nearest) distances for the time periods considered within this study, as obtained from a validated web-based calculator (Online Apogee/Perigee Ephemeris, 2013). The dates between phases need to be appropriately calculated along the elliptical path. Note that the lunar phase ephemeris really indicates the orientation of the earth-moon system with respect to the sun, and the lunar apogee/perigee ephemeris indicates when the moon is on the semi-major axis of its elliptical orbit about the earth; combination of the two would allow one to see how the lunar ellipse changes orientation over time (such as the snapshot given above in Figure 2).

Generic Ellipse and Variable Definition

It is possible to model the elliptical path of the moon by knowing only the successive apogee and perigee times and distances. Once that is done, one can interpolate along the ellipse using Kepler's Second Law. First, consider the ellipse drawn in Figure 3. All distance variables indicated on this figure are described in Table 4.

Both major and minor axes can be calculated from the perigee and apogee lengths, via the relations:

$$a = \frac{1}{2}(r_p + r_a) \quad , \quad b = \sqrt{r_p r_a}, \tag{1}$$

Figure 3: *Generic Ellipse. $A(\theta)$ represents the gray area swept out by angle θ.*

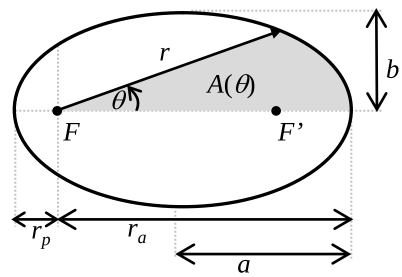


Table 4: *Meaning of Variables*

Variable	**Meaning**
F, F'	Focal points of ellipse.
a	Semi-major axis of ellipse.
b	Semi-minor axis of ellipse.
r_p	Shortest distance from ellipse to focal point F. ("Perigee" for the earth-moon system).
r_a	Longest distance from ellipse to focal point F. ("Apogee" for the earth-moon system).

Note that the meaning of r_p and r_a reverse if referenced to focal point F'.

$$r_p = a - \sqrt{a^2 - b^2} \quad , \quad r_a = a + \sqrt{a^2 - b^2}. \tag{2}$$

During this 1994-2004 study, the lunar perigee values (r_p) varied over 13,603 km from 356,571 km to 370,174 km whereas apogee values (r_a) only varied over 2,616 km from 404,088 km to 406,704 km (Online Apogee/Perigee Ephemeris, 2013). This variation is attributed to the gravitational interaction of the earth-moon-sun system.

The only other variable (not shown) which will appear in the calculations will be the ellipse's eccentricity, ε, which is a measure of the difference in length between the major and minor axes. It can be defined either by (a, b) or by (r_p, r_a):

$$\varepsilon = \sqrt{1 - \frac{b^2}{a^2}} = \frac{r_a - r_p}{r_a + r_p}. \tag{3}$$

If one takes F in Figure 2 as the earth's location and places the origin of a polar co-ordinate (r,θ) system at that location such that $\theta = 0$ corresponds to the horizontal line extending from F to F' (see Figure 3), then the ellipse can be described by the equation

$$r = \frac{a\left(1 - \varepsilon^2\right)}{1 - \varepsilon \cos\theta}. \tag{4}$$

Precession of Elliptical Orbit

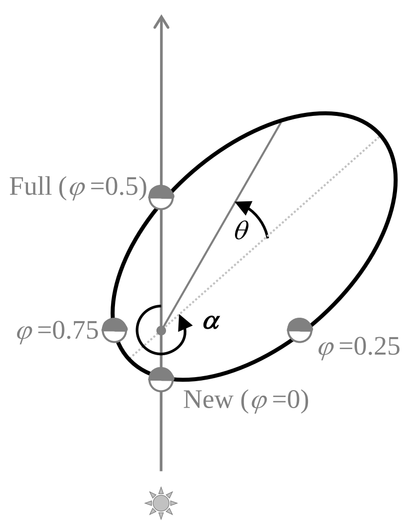
In this study, the lunar phase will be described by the variable φ whose values are positive fractions with $\varphi = 0$ (and 1) representing the new moon and $\varphi = 1/2$ representing full moon (see Table 5).

Table 5: *Values for Lunar Phase,* $\varphi \in [0, 1)$

Lunar Phase	φ	**Visibility** (local time)	**Peak** (local time)
New Moon	**0**	*not visible*	noon
Mid-Waxing Crescent	1/8	afternoon and post-dusk	3pm
First Quarter	1/4	afternoon to early night	6pm
Mid-Waxing Gibbous	3/8	later afternoon to early dawn	9pm
Full Moon	**1/2**	sunrise to sunset	midnight
Mid-Waning Gibbous	5/8	most of night to early morning	3am
3rd Quarter	3/4	late night and morning	6am
Mid-Waning Crescent	7/8	pre-dawn to morning	9am
New Moon	**1**	*not visible*	noon

The first (last) visible sliver seen after (before) the new moon occurs just after sunset (before sunrise).

Figure 4: *Alignment between earth-Moon system and earth-sun axis (lunar orbit exaggerated)*



Let the angle between the semi-major axis of the earth-moon system and the earth-sun axis be represented by the angle α (see Figure 4). The change in this angle over time represents the precession of the moon's orbit over time and can be gleaned from the ephemerides for lunar phases and apogee-perigee distances (NASA Lunar Phase Ephemeris, 2013; Online Apogee/Perigee Ephemeris, 2013). In order to construct a relation between the lunar phase φ, the moon's angle θ with respect to the semi-major axis of its orbit and the precession angle α, consider Figure 4 which shows the moon's ellipse at angle α with respect to the earth-sun line along with key lunar phases. From this figure, angles φ, θ, and α can be related to one another via:

$$\varphi = \frac{\theta + \alpha}{2\pi} + \frac{1}{2} - n, \tag{5}$$

where the integer n is introduced to ensure that $\varphi \in [0, 1)$. Note that one can rearrange this to yield

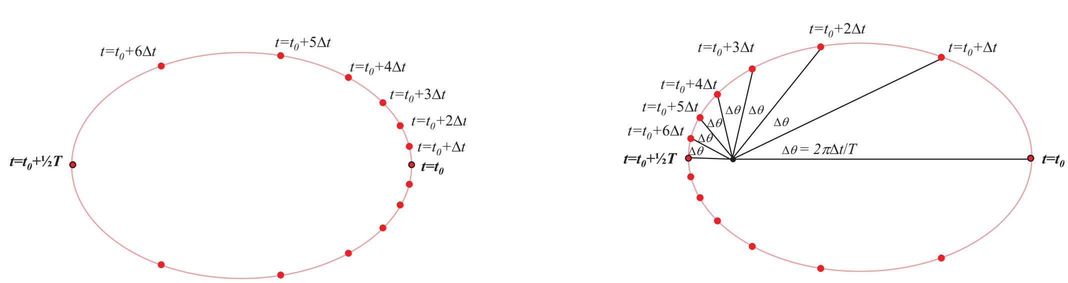
$$\alpha = 2\pi\left(\varphi - \frac{1}{2}\right) - \theta + 2\pi n \tag{6}$$

where again n is to ensure that $\alpha \in [0, 2\pi)$ for $\varphi \in [0, 1)$ and $\theta \in [0, 2\pi)$. However, in the analysis that follows n can remain implicit providing that if $\alpha(t_2) \geq \alpha(t_1)$ when interpolating between times $t_1 < t_2$ then 2π should be subtracted from $\alpha(t_2)$ and then re-added to the interpolated $\alpha(t_1 < t < t_2)$ should it be negative.

Interpolation Methods

The moon traveling in an elliptical path about the earth will travel faster near perigee and slower near apogee and hence will tend to spend more time furthest from the earth. Let the time that the moon reaches apogee ($\theta = 0$) be defined as $t = t_0$ and the time at the next perigee ($\theta = \pi$) be $t = t_0 + \frac{1}{2}T$, where T is the lunar period. Where is the moon at some intermediate time t between $t = 0$ and $t = t_0 + \frac{1}{2}T$? It might be tempting to interpolate θ linearly over time. By this method equal time intervals between t_0 and $t_0 + \frac{1}{2}T$ would yield equal angle intervals (see right-hand side of Figure 5), which is contrary to what we know of celestial mechanics (sampling equal time intervals would yield a clustering of positions near the apogee section since the moon is traveling slower there, see left-hand side of Figure 5).

Figure 5: *Equal Angle vs Equal Time Sampling*



Realistic Sampling of Orbital Motion (isochronal).

Erroneous Sampling via $\Delta\theta = 2\pi\Delta t/T$.

Thus, the correct interpolation would be to interpolate the area swept out between the known times, as this is exactly the relationship established between time and area according to Kepler's Second Law: a line segment drawn from the central object to the orbiting object sweeps out equal areas in equal times. Thus, the area swept out is proportional to the change in time. If it takes time $\frac{1}{2}T$ to sweep out upper half of the ellipse (the area of which is $\frac{1}{2}\pi ab$), then the area A swept out at time $t_0 < t < t_0 + \frac{1}{2}T$ would be:

$$\frac{A}{\frac{1}{2}\pi ab} = \frac{t-t_0}{\frac{1}{2}T} \implies A = \frac{\pi ab(t-t_0)}{T}. \tag{7}$$

The area at any given (r, θ) can also be determined from Equation (4):

$$A(\theta) = \int_{\theta_0}^{\theta} d\theta' \int_0^{r(\theta')} r' dr' = \frac{1}{2}\int r^2(\theta')d\theta' = \frac{1}{2}\int_{\theta_0}^{\theta} \frac{a^2\left(1-\varepsilon^2\right)^2}{\left(1-\varepsilon\cos\theta\right)^2}d\theta'.$$

After some simplification, this integration yields

$$A(\theta) = \frac{1}{2}\frac{b^2\varepsilon\sin\theta}{(1-\varepsilon\cos\theta)} + ab\tan^{-1}\left[\frac{a}{b}(1+\varepsilon)\tan\frac{1}{2}\theta\right] + \pi ab\,\mathcal{H}(\theta-\pi), \tag{8}$$

where $\mathcal{H}(\theta-\pi)$ arises from the appropriate integration constant and is the Heaviside function such that $\mathcal{H} = 0$ for $\theta \leq \pi$ and $\mathcal{H} = 1$ for $\theta > \pi$. The area described by Equation (8) is depicted in Figure 3. Unfortunately, since Equation (8) is transcendental in θ, no analytic solution for $\theta(A)$ exists. Therefore although one may compare Equations (7) and (8) to yield a function for $t(\theta)$ or $t(r)$, analytic forms for $\theta(t)$ and $r(t)$ do not exist and so iterative methods must be used to determine θ and r as a function of time. For this study, we used the bisection method, given that A monotonically increases with θ.

To interpolate values for the precession angle α, consider two consecutive phases within the lunar phase ephemeris φ_n and φ_{n+1}, occurring at times t_n and t_{n+1}, respectively. Once the corresponding angles θ_n and θ_{n+1} (respectively) are correctly determined at these times (as discussed above), the precession angles at these times (α_n and α_{n+1}) can also be calculated from Equation (6). Assuming that $\Delta\alpha \geq 0$ the precession angle at a time t between these phases ($t_n < t < t_{n+1}$) can be easily interpolated[1] (let $\Delta t \equiv t_{n+1} - t_n$, $\Delta\varphi \equiv \varphi_{n+1} - \varphi_n$ and $\Delta\theta \equiv \theta_{n+1} - \theta_n > 0$):

$$\frac{\alpha(t)-\alpha_n}{\Delta\alpha} = \frac{t-t_n}{\Delta t} \quad \Rightarrow \quad \alpha(t) = 2\pi\varphi_n - \pi - \theta_n + \frac{t-t_n}{\Delta t}\left(2\pi\Delta\varphi - \Delta\theta\right). \tag{9}$$

Note that (assuming that φ and θ are increasing) if φ represents a new moon then $\varphi_{n+1} = 1$ should be used and not $\varphi_{n+1} = 0$; similarly one needs to ensure that $\theta_{n+1} \geq \theta_n$ for these two phases. At this point, one needs to ensure that $\alpha \in [0, 2\pi)$. Once $\theta(t)$ and $\alpha(t)$ are computed for time t, Equation (5) can be used to establish the phase of the moon at that time.

Calculating Lunar Phase from Ephemerides

The following steps were used to determine when in the lunar phase cycle a specific event occurs. For each incident, let the time of incident be represented by t_i. To correlate the moon's position at the time of the incident, the following steps were taken:

[1] We leave the $2\pi n$ term implicit from here on, assuming that the algorithm used will catch any discontinuity discussed in the last paragraph of the last subsection.

1.1) From the apogee/perigee ephemeris, find the times of apogee and perigee which immediately precede and follow t_i. Denote these times as t_{ap-} and t_{ap+}. Note that t_{ap-} could refer to either an apogee time or a perigee time, depending on whichever immediately precedes the incident in time; whichever one it represents, t_{ap+} will represent the opposite extremum.

1.2) From the same ephemeris, note the apogee distance, r_a, and the perigee distance, r_p. Construct from these two variables the ellipse parameters a, b and ε, needed for Equations (4)-(8) and calculate the area $A(t_i)$ using Equation (7). Find the value of θ which yields the value of $A(t_i)$ in Equation (8). Denote this angle as θ_i. Finally note $r_i \equiv r(\theta_i)$ [Eq. (4)] is the earth-moon distance at the time of the incident.

2.1) From the lunar phase ephemeris, find the time of the lunar phase immediately preceding t_i. Denote that time as t_{ph-}. Denote this phase as φ_-. Following steps 1.1 and 1.2, calculate the moon's location at this time: r_{ph-} and θ_{ph-}, and the precession angle at this time: α_{ph-} [Equation (6)].

2.2) From the lunar phase ephemeris, find the time of the lunar phase immediately following t_i. Denote that time as t_{ph+}. Denote this phase as φ_+. Following steps 1.1 and 1.2, calculate the moon's location at this time: r_{ph+} and θ_{ph+}, as well as the precession angle at this time: α_{ph+} [Equation (6)].

2.3) Check for discontinuities in α; if α_{ph+} is near 0 and α_{ph-} is near 2π (such that $\alpha_{ph+} - \alpha_{ph-} > \pi$) then subtract 2π from α_{ph+}. Ensure that $\theta_{ph+} > \theta_{ph-}$. If it isn't, then add 2π to θ_{ph+}. Ensure that $\varphi_{ph+} > \varphi_{ph-}$. If it isn't, then set $\varphi_{ph+} = \varphi_{ph+} + 1$.

2.4) Use Equation (5) with Equation (9) to determine the lunar phase at time t_i:

$$\varphi(t_i) \quad = \quad \left\{\varphi_{ph-} + (t_i - t_{ph-})\frac{\Delta\varphi}{\Delta t}\right\} + \frac{1}{2\pi}\left\{\theta_i - \left[\theta_{ph-} + (t_i - t_{ph-})\frac{\Delta\theta}{\Delta t}\right]\right\}. \quad (10)$$

Note that the first term in Equation (10) is simply the linear interpolation between successive phases. The second term is a correction factor representing the difference between the computed angle at time t_i and an angle linearly interpolated between the times t_{ph-} and t_{ph+} (preceding and successive phases, respectively). If it were possible to linearly interpolate between successive θ (which was demonstrated above to be incorrect), then this term would vanish and $\varphi(t_i)$ could simply be calculated by linearly interpolating between the previous and the next lunar phase.

Probability Densities for Lunar Range, r

In the "Analysis" section, some of the statistical analyses will make use of explicit distribution functions for r which will be derived here. Although an analytic form for $r(t)$ is not achievable, an analytic form for the distribution function for r, namely $p_r(r)$, can be derived.

Let us begin with the statement that any random event which is independent of the lunar motion will occur at any time during the moon's orbital period with equal probability.

The distribution function for time is therefore the uniform distribution $p_t(t) = T^{-1}$ for $0 \leq (t - t_0) \bmod T \leq T$ (where t_0 is chosen to be some reference time in the past such that $\theta(t_0) = 0$). From this we can use Kepler's Second Law [Eq. (7)] and the chain rule for probability distribution functions to determine the probability distribution function for lunar distance r:

$$p_r(r) = 2p_t(t(A(\theta(r))))\frac{dt}{dA}\frac{dA}{d\theta}\left|\frac{d\theta}{dr}\right| = \frac{r}{\pi a\sqrt{(a^2-b^2)-(r-a)^2}}, \tag{11}$$

which is consistent with the solution given by Kessler (1990). Note that the factor of two, along with the absolute sign, arises since r monotonically decreases for $0 \leq \theta \leq \pi$ and monotonically increases for $\pi \leq \theta \leq 2\pi$. Thus, an absolute sign is used to ensure $p_r(r) \geq 0$ and the factor of two arises from mapping of $\theta \in [0..\pi]$ and $\theta \in [\pi..2\pi]$ to the same values of r. Finally, the cumulative probability function, $P(R \leq r)$ can be easily derived:

$$\begin{aligned} P(R \leq r) &\equiv \int_0^r p_r(r')dr' \\ &= 1/2 - \frac{1}{\pi a}\sqrt{(a^2-b^2)-(r-a)^2} + \frac{1}{\pi}\tan^{-1}\left(\frac{r-a}{\sqrt{(a^2-b^2)-(r-a)^2}}\right). \end{aligned} \tag{12}$$

The first term is the integration constant set to 1/2 in order to satisfy $P(R \leq r_p) = 0$. With this choice, it is easy to verify that $P(R \leq r_a) = 1$. For the eleven year window, the perigee and apogee distances vary greatly with each cycle. Therefore in order to construct a cumulative probability reflective of this, we use

$$\tilde{P}(R \leq r) = \frac{1}{N_{ho}}\sum_{i=1}^{N_{ho}} P_i(R \leq r), \tag{13}$$

where P_i is Equation (12) using the ephemeris r_a and r_p values for the i^{th} half-orbit (e.g., perigee-to-apogee or apogee-to-perigee) and N_{ho} is the total number of half-orbits within that eleven-year window.

One may now easily compute the probability of the earth-moon distance to be within the distances r_1 and $r_2 > r_1$ via

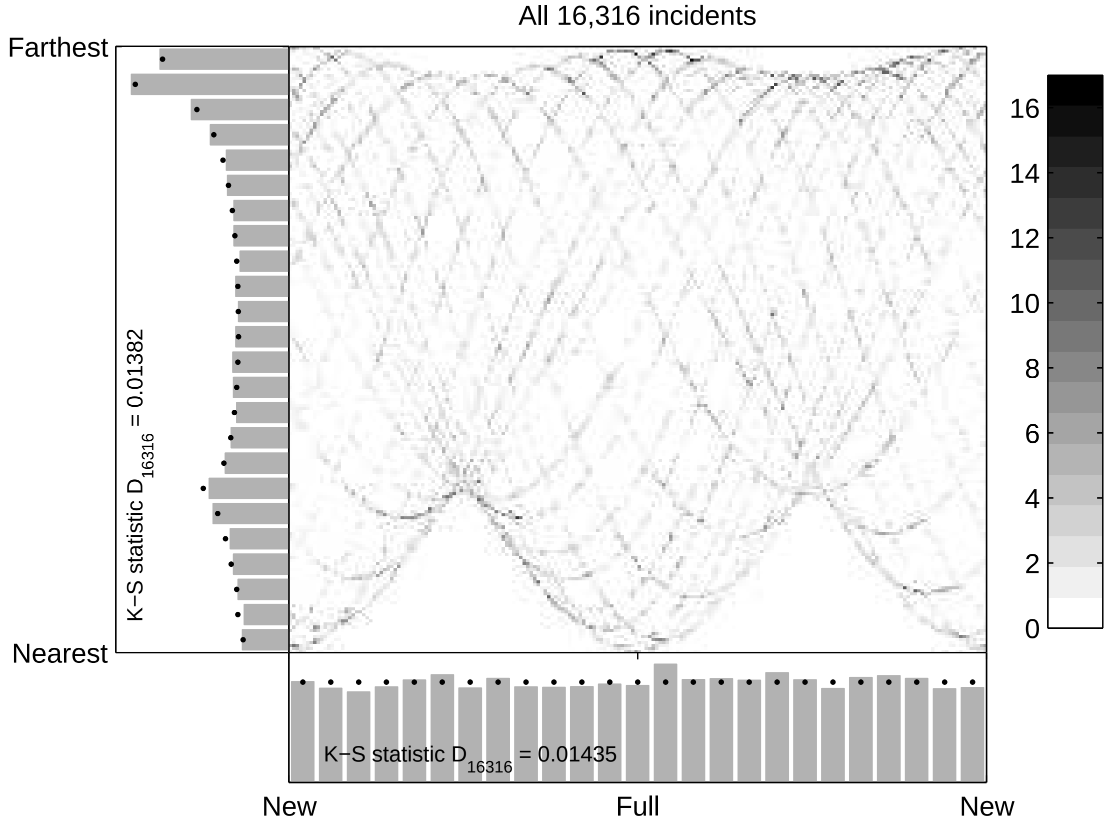
$$\tilde{P}(R \leq r_2) - \tilde{P}(R \leq r_1). \tag{14}$$

Analysis

For each SAR datum within the data set, the lunar phase and lunar distance were calculated. Figure 6 depicts these data in a two-dimensional heat map (number of incidents as a function of lunar phase and lunar distance) with histograms at the bottom and to the left to depict each dimension's distribution of data.

In this figure, there are 24 vertical bins and 25 horizontal bins for the histograms surrounding the heat map in Figure 6; the choice of bin size is derived from the optimal bin width equation of Scott (1979) for data sets which do not necessarily have an underlying Gaussian distribution. However, the heat map itself has a 200 by 200 bin density in order

Figure 6: *Number of SAR Incidents as a Function of Lunar Phase and Lunar Distance.*



Note that the distribution of incidents over lunar phase is summarized in a histogram along the bottom and the distribution of incidents over lunar distance is summarized in a histogram along the left hand side. Black dots in the two histograms are the theoretical expected probabilities for each bin, derived from Equation (14).

to show the relationship between lunar distance and lunar phase as indicated by the orbital patterns seen therein. On every histogram's bar is a dot which represents the likelihood of an incident occurring within the values bounded by the bar based on theoretical distributions. For the lunar distance histogram on the left, the probabilities were determined by Equation (14). For the lunar phase histogram at the bottom, the probabilities are derived from a uniform distribution across all phase values.

The dots are included for a visual comparison of theoretical values to the actual plotted data; that is, the gray bars represent the percentage of data which exist within the bin's range whereas the black dot represents what percentage should be within that range if the data truly follows the distribution function (such as from Equation (14) for the vertical histogram). Although it is tempting to apply a chi-squared test to see how well they agree, we instead performed a set of non-parametric Kolmogorov-Smirnov statistical tests of the data against the theoretical cumulative probability function (Equation (13) for the left-sided histogram and $y = x$ for the bottom histogram). The Kolmogorov-Smirnov statistic, D_n provides a powerful goodness-of-fit test when the underlying cumulative probability function is continuous and contains no parameters which need determining from the sampled data

(Stephens, 1974; Law & Kelton, 2000): circumstances which are precisely what we have here. The values of D_n are shown in the histograms labeled as "K-S statistic $D_{...} = ...$" The values for the Kolmogorov-Smirnov statistic within each SAR class are provided in Table 6 for lunar distances and in Table 7 for lunar phase.

Table 6: *Kolmogorov-Smirnov statistic (D_n) for lunar distance correlation within each SISAR class.*

	In Distress	Potential Distress	Non-Distress	False Alarms and Hoaxes	
Aeronautical	0.06762	0.05999	0.1318	0.02879	*0.02184*
Maritime	0.02892	0.02193	0.01798	0.03622	*0.01967*
Humanitarian	0.05395	0.07413	0.03835	0.08017	*0.03984*
Unknown	-	-	-	0.02422	
	0.02942	*0.02172*	*0.01653*	*0.01011*	*0.01382*

Grey italic numbers are the test statistics for mutually exclusive analyses of the roll-up data across rows and columns.

Table 7: *Kolmogorov-Smirnov statistic (D_n) for lunar phase correlation within each SISAR class.*

	In Distress	Potential Distress	Non-Distress	False Alarms and Hoaxes	
Aeronautical	0.03406	0.063	0.08199	0.01578	*0.01046*
Maritime	0.03088	0.02686	0.02137	0.02531	*0.01935*
Humanitarian	0.04123	0.03013	0.03255	0.02419	*0.02511*
Unknown	-	-	-	0.03039	
	0.02689	*0.02672*	*0.01918*	*0.008534*	*0.01435*

Grey italic numbers are the test statistics for mutually exclusive analyses of the roll-up data across rows and columns.

Stephens (1974) demonstrated that in cases in which the test cumulative probability function is completely known, a modified test statistic D_n,

$$\bar{D}_n \equiv \left(\sqrt{n} + 0.12 + \frac{0.11}{\sqrt{n}}\right) D_n \tag{15}$$

should be used since a single vector, c_{ci} (where ci is a confidence interval), rather than a matrix, can be used to compare against $\bar{D}_n$ to test the null hypothesis (Law & Kelton, 2000). That is, we can reject the null hypothesis that the SAR data are independent and identically distributed if $\bar{D}_n > c_{\mathsf{ci}}$. Tables 8 and 9 depict the modified Kolmogorov-Smirnov statistic for each of the SISAR class. In each table along the bottom and to the right are italic numbers in gray. They represent the modified Kolmogorov-Smirnov statistic for the entire column and row (respectively). What we found was for each SISAR class (e.g., "Maritime, In Distress", "Maritime, False Alarms", etc.) we could only reject the null hypothesis that no correlation exists between the SISAR data and the lunar phase/distance for the maritime non-distress SAR incidents at a 99% confidence level. That is, this particular subset of SAR incidents seem to have a correlation with lunar phase and distance. Note that since this particular SISAR category is more abundant than any other by almost an order of magnitude (see Table 3), we also saw the rejection of the null hypothesis for the set of all non-distress incidents in the lunar phase and the set of all maritime incidents for both lunar phase and distance; all because a majority of those incidents are maritime non-distress. The values of $\bar{D}_n$ which led to the rejection of the null hypothesis are indicated in bold in Tables 8 and 9.

Table 8: *Modified Kolmogorov-Smirnov statistic ($\bar{D}_n$) for lunar distance correlation within each SISAR class.*

	In Distress	Potential Distress	Non-Distress	False Alarms and Hoaxes	
Aeronautical	1.275	0.6623	1.289	1.114	*0.9919*
Maritime	0.7693	0.6021	**1.63**	1.61	***2.122***
Humanitarian	1.102	1.148	0.9291	1.375	*1.558*
Unknown	-	-	-	0.8181	
	1.129	*0.7237*	*1.559*	*0.7075*	***1.767***

Grey italic numbers are the test statistics for mutually exclusive analyses of the roll-up data across rows and columns. Bold values indicate instances where the null hypothesis is rejected.

Table 9: *Modified Kolmogorov-Smirnov statistic ($\bar{D}_n$) for lunar phase correlation within each SISAR class.*

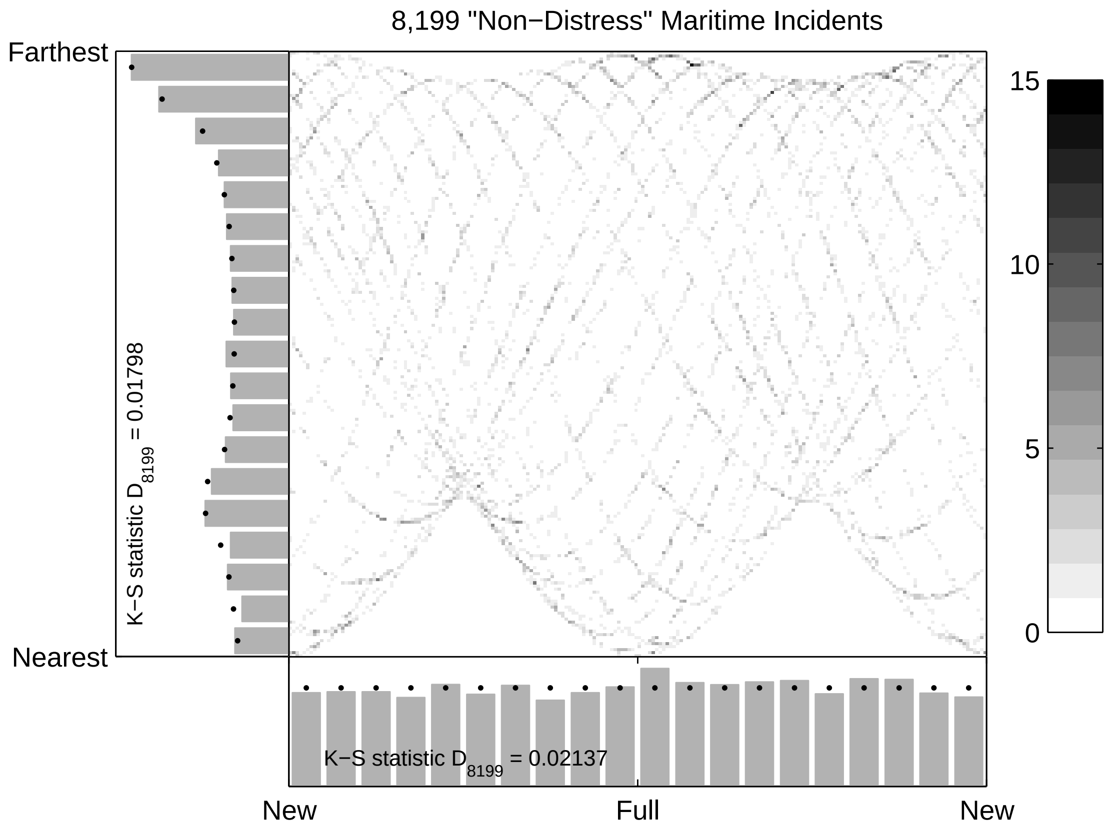
	In Distress	Potential Distress	Non-Distress	False Alarms and Hoaxes	
Air	0.6423	0.6955	0.8015	0.6109	*0.4753*
Maritime	0.8216	0.7375	**1.938**	1.125	***2.088***
Humanitarian	0.8421	0.4667	0.7885	0.415	*0.9816*
Unknown	-	-	-	1.027	
	1.032	*0.8901*	***1.809***	*0.597*	***1.835***

Grey italic numbers are the test statistics for mutually exclusive analyses of the roll-up data across rows and columns. Bold values indicate instances where the null hypothesis is rejected.

Maritime Non-Distress SISAR Class

We concentrated further analysis on the class of maritime non-distress incidents as it is the only one which has a correlation with lunar phase and distance. The heatmap and histograms associated with this subset of data are found in Figure 7. The univariate categorical data set for lunar phase was grouped into the four bins "new", "first quarter", "full" and "third quarter" (see Table 10) using mid-waxing/mid-waning phases as the boundaries (see Table 5). The accompanying counts of SAR incidents are shown in the third column of Table 10 as well as the plot shown in Figure 8.

Figure 7: *Number of SAR Incidents as a Function of Lunar Phase and Lunar Distance for All Maritime Non-Distress Incidents.*



Note that the distribution of incidents over lunar phase is summarized in a histogram along the bottom and the distribution of incidents over lunar distance is summarized in a histogram along the left hand side. Black dots in the two histograms are the theoretical expected probabilities for each bin, derived from Equation (14).

When encountering data such as this, inference can be drawn using a Chi-square contingency table. We used the principle of parsimony to continue to use the simple model that the mean number of incidents in the four time periods are independent of lunar phase. The contingency table (Table 10) includes in the final column the Pearson's chi-square test statistic χ^2 (Agrestri, 2007), determined by

$$\chi^2 = \sum_{i=1}^{4} \frac{(O_i - E_i)^2}{E_i}, \tag{16}$$

where E_i is the expected number of counts within the i^{th} time period.

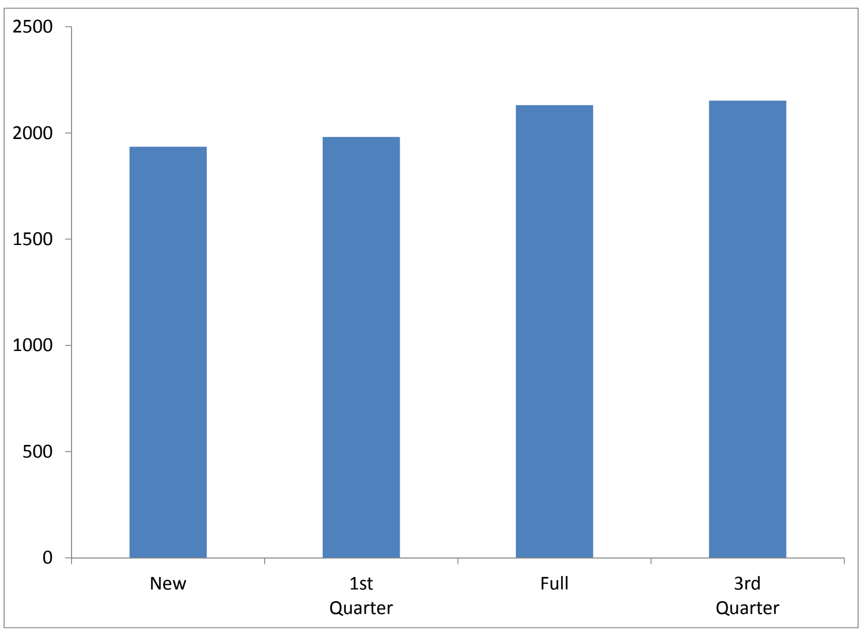
The calculated χ^2 needs to be compared to a critical value of χ^2, which depends upon the number of degrees of freedom in Table (10), which is "# of rows" - 1 = 3. If a test of significance (and its associated test statistic) gives a p-value lower than or equal to the significance level, the null hypothesis is rejected. The lower the significance level chosen, the stronger the evidence required. Convention (Stigler, 2008; Fisher, 1925) suggests the analysis is conducted at an uncertainty level of $100 - 95 = 5\%$. The value of χ^2 at an

Table 10: *Grouping Maritime non-distress SAR Incidents into Four Lunar Phase Bins*

φ-Group	Nearest Lunar Phase	# of Incidents (O_i)	$\frac{(O_i - E_i)^2}{E_i}$
$\Delta\varphi_{\text{new}} \equiv \varphi \in \left[0, \frac{1}{8}\right) \cup \left[\frac{7}{8}, 1\right)$	New Moon	1,935	6.42
$\Delta\varphi_{1/4} \equiv \varphi \in \left[\frac{1}{8}, \frac{3}{8}\right)$	First Quarter Moon	1,981	2.30
$\Delta\varphi_{\text{full}} \equiv \varphi \in \left[\frac{3}{8}, \frac{5}{8}\right)$	Full Moon	2,131	3.22
$\Delta\varphi_{3/4} \equiv \varphi \in \left[\frac{5}{8}, \frac{7}{8}\right)$	Third Quarter Moon	2,152	5.10
		$\chi^2 = 17.05$	

For each group, the expected # of incidents is $E_i = 8,199/4 = 2,049.75$

Figure 8: *Number of SAR incidents for each of the four values of* $\Delta\varphi$.



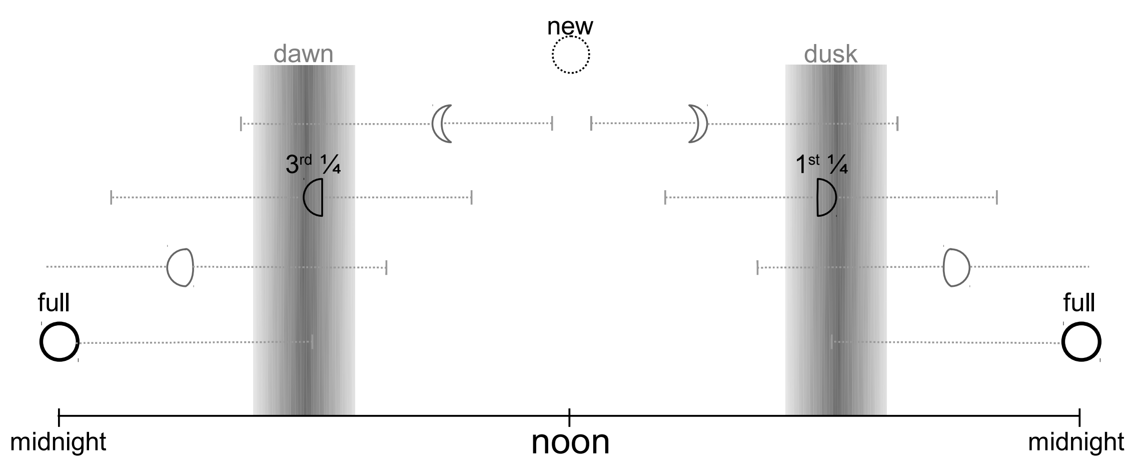
uncertainty level of 5% and three degrees of freedom is 7.8. Since the calculated value for χ^2 is larger than the critical value for χ^2, the null hypothesis that the number of SAR incidents is independent from lunar phase is rejected.

What is intriguing about Figure 8 is that we see a correlation not with a particular phase, but with a *range* of phases. In order to explore this, we combine the four lunar bins into two ($\Delta\varphi_{\text{new}} + \Delta\varphi_{1/4}$ and $\Delta\varphi_{\text{full}} + \Delta\varphi_{3/4}$) and then divide each new bin into whether their occurrence was in the pre-dawn, daytime or post-dusk portions of the day (based on sunrise and sunset times at the incident location) and whether or not the moon was in the sky (based on the moonrise and moonset times at the incident location). The results are shown in Table 11, and Figure 9 is provided to help visually illustrate when the four phases occur during the day.

Table 11: *Breakdown of Maritime Non-Distress Incidents*

	Pre-Dawn		Daytime		Post-Dusk	
	$\Delta\varphi_{new}$ $+\Delta\varphi_{1/4}$	$\Delta\varphi_{full}$ $+\Delta\varphi_{3/4}$	$\Delta\varphi_{new}$ $+\Delta\varphi_{1/4}$	$\Delta\varphi_{full}$ $+\Delta\varphi_{3/4}$	$\Delta\varphi_{new}$ $+\Delta\varphi_{1/4}$	$\Delta\varphi_{full}$ $+\Delta\varphi_{3/4}$
Moon In Sky	6	94	2271	697	30	246
Moonless	98	24	707	2519	804	703
Total	104	118	2978	3216	834	949

Figure 9: *Position of moon at its meridian for various phases. Grey lines notionally indicate the extent to which each phase above the horizon.*



For pre-dawn incidents, it is far more likely to see the moon in its full or 3rd quarter phase, which is reflective in Table 11. What is curious about this table, though, is that there are far more incidents occurring in the post-dusk times during full-to-3rd-quarter phases despite early evening being the typical time to see the first quarter phase.

Conclusions and Discussion

We examined 16,315 SAR incidents from an 11 year period (1994-2004) spanning much of Canada and looked to see if there was any correlation with either lunar phase or lunar distance. Any correlation, if significant, would have the potential to influence the planning of ready-to-respond times for SAR. We used celestial mechanics and established ephemerides to carefully assign the specific lunar phase and distance to each event and conducted a statistical analysis on each of these two sets of measures. We found no correlation with any particular lunar phase, which seems to corroborate many of the previous "lunacy" studies wherein no correlation was found with the full moon. There are a few correlations, however, that did arise and deserve attention here.

First of all, Figures 6 and 7 indicate more SAR incidents occurring when the moon was farthest away from the earth. This is a wonderful example of "correlation" versus "causation". Quite simply, because the moon spends more time further from the earth than near it, there will be more random events occurring when the moon is farther away. That is, when examining the histograms to the left of the heat maps for both of these figures, one should expect a larger number of random incidents at larger lunar ranges. This hypothesis was validated by deriving the expected distribution of random events as a function of lunar distance and comparing it against the data set. The data are correlated, but not causal. That is, data clusters at larger lunar ranges simply because the moon spends more of its time there, rather than the moon having a greater influence on terrestrial events when farther away. In fact, it would be of interest to re-examine some of the previous studies (e.g., Lieber's study (1978)) which claim the "lunacy" effect is tied to the lunar tidal force. If that were the case, then one would see more incidents occurring when the moon is closer to the earth when the lunar tidal force is the strongest. However, if those data sets are like the data we examined, and were indeed independent of the lunar distance, one should expect to see more of those events occurring when the moon was farther from the earth when its lunar tidal force is the weakest (tidal forces are inversely proportional to distance cubed). Thus, the lunar distance should be used in these kinds of studies in order to test the validity of there being a tidal influence.

Secondly, for a large set of the SAR data that we examined, there was no correlation between the incidents and the lunar phases. However, for a large subset (8,199 incidents) we did find a correlation with a *range* of phases. More of the maritime non-distress SAR incidents seem to have occurred in the full moon and 3rd quarter phases. Although it is not clear from the data set why this was the case, we conjecture that it may have been something to do with the amount of extra light that these phases provide which may lead to increased boating at night. This would be in line with other studies (Barr, 2000; Baxendale & Fisher, 2008), which indicate that the increase in nighttime illumination might be the influencing factor (although these previous studies discussed light-sensitive disorders whereas we are merely suggesting here increased water traffic due to 50% (or more) moonlight). However, we cannot definitively conclude this since the SISAR data do not include all water traffic at that time (only those needing assistance) from which an overall increase in traffic might be observed. In any event, over the 11 year period, the full moon window and the three-quarters window saw an increase of 367 incidents over the new moon window and first-quarter window. This only constitutes 4.5% of non-distress maritime incidents and only 2.25% of all SAR incidents. As such, it would seem unlikely to be a strong motivation to alter SAR ready-to-respond postures; rather than the four incidents per day that one may see on average, there may be one or two additional incidents over the two week period in question.

Finally, even with a noted correlation of the non-distress maritime class of incidents with a range of phases, it ought to be stressed that there was no correlation found between lunar phase and any other SISAR maritime classification, including both "in distress" and "potential distress". Indeed, no such correlation was found with any of the other SISAR classes (the entire aerospace class, humanitarian class and unknowns, the last of which is a collection of both hoaxes and false alarms). What is more, in none of the data was there a correlation with any particular lunar phase, such as the full moon. This leads to some interesting conclusions. Either the full moon exerts a negative psychological influence on human behavior but never in a way that generates incidents requiring search and rescue help, or the SAR data indicate that there is no such influence from the full moon. Given the large span in time (11 years) and geographical span of the comprehensive SISAR data examined here, it is hard not to side with the latter conclusion.

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